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Comparison of Least Square Fitting Algorithms for the Evaluation of Roundness Error

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Abstract

Roundness error is one of the significant quantifiers used in quality control of cylindrical or circular components. In both laboratories and industries, direct estimation of roundness error is predominantly obtained using the Coordinate Measuring Machine (CMM) and/or form testing device. Estimation of roundness error can be accomplished using four different reference circles viz, Minimum Circumscribed Circle (MCC), Maximum Inscribed Circle (MIC), Minimum Zone Circle (MZC) and Least Square Circle (LSC). MZC matches closely with ISO standards of roundness, while, MCC is used for roundness evaluation of shafts and MIC for holes. LSC is a very popular and widely used approach owing to its robustness, computational efficiency, and its ability to work on a wide range of datasets. Different LSC algorithms have been developed which fall into two broad categories: geometric fits and algebraic fits. The performance of these algorithms, in terms of their accuracy and computational time, is influenced by the type and size of input data. A test for the relative performance of these algorithms is necessary in order to make a decision about which algorithm is to be used for a particular application. In this paper, a few selected geometric fits and algebraic fits used for LSC fitting have been compared for different datasets. In order to quantify its ability to accommodate varied data (CMM and form profile), the performance of all the algorithms is compared using the same ten datasets measured using both devices.

Keywords: Roundness Error, Least Square Circle, Coordinate Measuring Machine, form data, metrology

1. INTRODUCTION

Circular components account for about three-quarters of all engineering components making their prominence span across many different industries. Circularity/roundness is a measure of the degree of compliance of a component to the profile of an ideal circle. Approaching perfect circularity is an idealized situation because every manufacturing process entails some degree of inaccuracy. Few of those inaccuracies which attribute to out-of-roundness include clamping distortions, spindle run-out, temperature change, erratic cutting etc. Thus, assembly of any circular component necessitates both dimensional and geometrical tolerances to be met as per the requirements.

ISO 1101 [1] defines roundness as the radial distance between two concentric circles that are separated by minimum possible distance and contains all the measured points on the profile. Roundness evaluation is predominantly carried out using four different reference circles viz., Minimum Circumscribed Circle (MCC), Maximum Inscribed Circle (MIC), Minimum Zone Circle (MZC) and Least Square Circle (LSC). MZC is the reference circle which complies most closely with the standard definition of roundness. It gives the minimum value of roundness error when compared to the other reference circles; it is widely preferred. MCC and MIC references are used for roundness error estimation of shafts and holes respectively to follow the industrial practices [2]. Among these four, LSC is the one most commonly adopted for industrial applications. It is based on the mathematical principle of minimization of the sum of the square of deviations of the measured points from the fitted circle. LSC method is robust, computationally efficient and can easily accommodate large-sized datasets, thus justifying its wide usage. With each of the reference circle having its own inherent perks, the choice of their usage for roundness evaluation depends upon the specific application.

LSC fitting algorithms [3] are categorized into two types – geometric fits and algebraic fits. Geometric fits are iterative and involve intensive computations while maintaining high accuracy. This leads to higher computational time and probability of divergence due to its iterative nature. Algebraic fits are simpler, reliable and non-iterative in nature. Hence, they are preferred in cases of large datasets and to provide an initial guess for the subsequent geometric fitting procedure. The accuracy, as well as the computational efficiency of the algorithms depend on the type and size of the data input. The selection of appropriate LSC algorithm for a particular application must be based on the tradeoff between the degree of accuracy required and the computational time. Once an algorithm is selected, the LSC circle can be fitted to the dataset to obtain the center, say O. Two concentric circles with O as their center are constructed so as to pass through the farthest point and the nearest point calculated from the LSC center. The difference between the maximum and minimum radius gives the required roundness error.

Coordinate Measuring Machine (CMM) and form tester device are widely used to evaluate the roundness error by sampling points from the test component. Such machines have options to evaluate roundness error using all the four reference circles. Each of the reference circle has many different approaches/techniques for its method of computation. It is essential for a machine developer to select the best possible algorithm for each, in order to provide the most accurate result. This entails a comparison of computational accuracy among the many available circle fitting approaches. The present work deals with the comparative study of different LSC computation algorithms owing to its wide usage in many roundness measuring machines. The objective of the present work is to compare a few LSC fitting algorithms, both geometric and algebraic fits, and evaluate their accuracy of roundness error computation.

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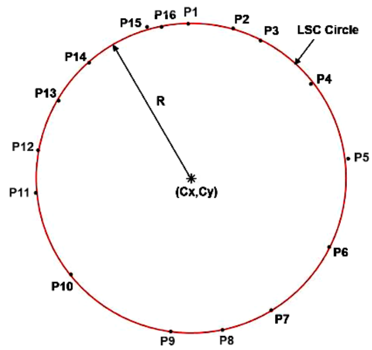
A total of ten algorithms have been chosen from the literature for the performance comparison. The selected algorithms for comparison include Spath Circle [4], Trust Region Method [5], Levenberg-Marquardt algorithm [6], Landau Fit [7], Reimann Circle [8], Kasa Circle [9], Pratt Circle [10], Taubin Circle [11], Hyper Fit [12] and Kukush-Markovsky-Van Huffel (KMvH) Algorithm [13]. CMM data and form tester data have different properties in terms of the extent of undulation in their profile. This difference in the nature of the two types of data can be seen from Fig 1. Fig 1a shows the sample profile obtained from CMM which is fitted with LSC circle (using Levenberg-Marquardt algorithm) characterized by radius R and center (Cx, Cy). Fig 1b shows the form tester data for one of the test pieces which was manufactured for this study. Thus, to study the ability of LSC algorithms to work on different types of data, the presented comparison has been done using ten different datasets each obtained using CMM and roundness tester.

2. METHODOLOGY

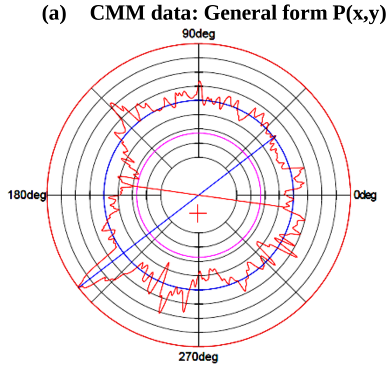
The relative superiority in the accuracy of roundness error computation among the ten algorithms was established as follows. The study was conducted by inputting the same sets of data (CMM and form profile) to all ten benchmark algorithms. For a particular dataset, the algorithm(s) which gave the smallest magnitude of roundness error value was found. Ten different datasets each for CMM and form data were used. For comparison of CMM data, seven sets were obtained through measurement from test pieces manufactured for this study and three published datasets [14-16] were used. In the case of form data, all ten datasets were extracted through measurement. In all, ten LSC algorithms were compared against ten different datasets for CMM and form data separately.

Upon obtaining the results of the 100 test runs for CMM and form data individually, each of the ten algorithms is given its performance score as elucidated. Firstly, the tabulated roundness errors are compared column-wise, and the row with the least value is highlighted. This helps in finding the most accurate roundness algorithm for a given dataset and the process is repeated for all ten sets of data. If there are multiple rows along a single column giving the same lowest value, all are selected. Next, conducting a row-wise comparison brings out the best-performing algorithm. For this, among the ten datasets input, the number of datasets from which the algorithm gave the least roundness error value is counted using the total number of highlighted boxed in a particular row. This total number corresponds to the score assigned to an algorithm. For instance, when an algorithm 'ABC' has four highlighted blocks along its row, it signals that ABC gave the most accurate answer among the ten algorithms for 4 out of 10 data inputs. This scoring technique serves as a direct indicator for their relative accuracy in roundness computation. The algorithm(s) which has the maximum numerical score have been forwarded for use.

For this comparative study, all the ten algorithms have been replicated using MATLAB. It was found from the literature survey of benchmark algorithms that different authors have used disparate software platforms to model their algorithms. Using a common platform for testing is mandatory to eliminate computation errors caused due to different compilers. The usage of different computational platforms has more perturbing effects in the case of form data where the number of iterations is large. These small errors are of importance because the magnitude of roundness error values is generally in micrometer order. Hence, even trifling changes in the third or fourth decimal place is of relevance in such cases of roundness comparisons.



(a) CMM data: General form P(x,y)



(b) Form tester data: General form P(r,θ)

Fig 1. Sample profiles from roundness evaluation machines

3. EXPERIMENTAL METHODS

Seven low carbon steel cylindrical test pieces were manufactured for this comparative study to collect form and CMM data. The diametrical range of the test pieces was from 20 to 110 mm. This particular interval was chosen to cover the three popularly used cutoff values of 50, 150 and 500 undulations per revolution used in form tester devices according to ISO 12181-2 [17]. The following were the dimensions of the 7 workpieces: 20 mm, 32 mm, 42 mm, 48 mm, 56 mm, 76 mm, and 110 mm. The test pieces were designed such that these dimensions included both classes of the convex and concave profile. The configuration of the test piece surfaces varies as both inner (32 mm, 42 mm, 56 mm) and outer curved surfaces (20 mm, 48 mm, 76 mm, 110 mm) This was done to mimic the shafts and holes which are frequently tested for roundness in the industries. The different configurations would create disparate form profiles creating an equal platform for testing the algorithms.

3.1 CMM data acquisition

The datasets for CMM were measured using the Mitutoyo Bright-A 504. It has a measurement accuracy of 4 +5 L/1000 µm and repeatability of 3.0 µm. Renishaw's ruby tip PH10T probe was used for all measurements. The workpiece datum was established using the 'surface and two-circle method' and the coordinates were measured for each of the seven workpieces at a particular cross-section. Systematic errors were reduced by using proper initial probe calibration. The datasets size range from 16 to 32 points. The distribution of the data points over the surface has random angular intervals but caution was taken to distribute the points over all four quadrants.

Table 1. Comparison of benchmark algorithms using CMM data

ALGORITHMS	1	2	3	4	5	6	7	Ref[14]	Ref[15]	Ref[16]
Hyper	0.3741	0.9530	0.4264	0.8420	0.6568	0.6058	0.9036	38.1640	2.6188	0.0298
Kasa	0.3742	0.9530	0.4264	0.8417	0.6571	0.6058	0.9036	38.1985	2.6069	0.0298
KMvH	0.3738	0.9530	0.4264	0.8419	0.6573	0.6056	0.9036	38.1526	2.6192	0.0341
Landau	0.3740	0.9518	0.4262	0.8420	0.6566	0.6058	0.9036	38.1738	2.5961	0.0298
LMA	0.3740	0.9517	0.4260	0.8422	0.6561	0.6057	0.9037	38.1734	2.5961	0.0298
Pratt	0.3741	0.9430	0.4264	0.8420	0.6568	0.6058	0.9036	38.1640	2.6188	0.0298
Riemann	0.3741	0.9530	0.4264	0.8419	0.6569	0.6058	0.9036	38.1764	2.6146	0.0298
Spath	0.3740	0.9518	0.4262	0.8420	0.6566	0.6058	0.9036	38.1737	2.5961	0.0298
Taubin	0.3741	0.9530	0.4264	0.8420	0.6568	0.6058	0.9036	38.1633	2.6190	0.0298
Trust Region	0.3740	0.9517	0.4260	0.8422	0.6561	0.6057	0.9037	38.1734	2.5961	0.0298

Table 2. Comparison of benchmark algorithms using form tester data

ALGORITHMS	1	2	3	4	5	6	7	8	9	10
Hyper	6.2341	49.1427	31.6873	9.4646	36.4976	11.0093	6.9126	4.2348	6.3379	62.5290
Kasa	6.1254	49.4023	31.6741	9.4867	36.4869	11.0175	6.8607	4.2196	6.3291	62.6189
KMvH	6.2374	49.1428	31.6874	9.4633	36.4973	11.0031	6.9130	4.2347	6.3381	62.5230
Landau	5.9286	49.7211	31.6483	9.3804	36.4820	11.1074	6.7950	4.2075	6.3569	64.1801
LMA	5.9284	49.7214	31.6484	9.3801	36.4821	11.1088	6.7949	4.2076	6.3570	64.1805
Pratt	6.2341	49.1427	31.6873	9.4646	36.4976	11.0093	6.9126	4.2348	6.3379	62.5290
Riemann	6.1957	49.2351	31.6825	9.4725	36.4937	11.0122	6.8941	4.2294	6.3348	62.5613
Spath	5.9286	49.7211	31.6483	9.3804	36.4820	11.1074	6.7950	4.2075	6.3569	64.1801
Taubin	6.2370	49.1378	31.6875	9.4644	36.4977	11.0092	6.9136	4.2350	6.3381	62.5266
Trust Region	5.9284	49.7214	31.6484	9.3802	36.4821	11.1088	6.7948	4.2076	6.3570	64.1805

Unit: µm

Table 3. Scores obtained by benchmark algorithms

Data	Hyper	Kasa	KMvH	Landau	LMA	Pratt	Riemann	Spath	Taubin	Trust Region
CMM	2	3	4	3	5	2	2	3	2	5
Form	0	1	2	3	2	0	0	3	1	2

3.2 Form tester data acquisition

Mitutoyo RA1600M form tester was used to obtain the ten form datasets using 12AAL021 stylus, carbide ball type of size ø1.6 mm. The same seven low carbon steel workpieces were used for data acquisition. Following ISO 12181-2, filter selection and cutoff ranges were made. Datasets sizing up to 1800 points were obtained from each test piece, but, for the ease of illustration, results of datasets of size 72 points have been discussed in this paper. Measurement using form tester always entails the problem of limacon formation which results from improper centering. Centering in a form tester is the process of matching the center of the workpiece with the machine center. More the deviation between the two centers, more is the limacon error. Even if the LSC algorithms have the ability to match the real roundness error closely, the presence of limacon error may distort the computed roundness error. Thus, minimization of this error would reduce the difference between the real and estimated roundness value. Thus, centering was done using the in-built Digital Adjustment Table guidance system. Though limacon error cannot be completely eliminated, this inbuilt function helps minimize it to a great extent.

4. RESULTS AND DISCUSSION

The results of the comparative study for CMM and form data are presented in Table 1 and Table 2 respectively. Table 1 shows that the algebraic fits such as Kasa, Pratt, Taubin and Hyper Fit algorithms are comparatively less accurate in case of CMM data. In the case of form data, a similar trend is found in the performance of algebraic fits. It is observed from Table 2 that the Trust Region Method and Levenberg-Marquardt algorithm have the highest score for CMM data and, Landau Fit and Spath Circle show the best performance for form data. All these four approaches belong to the category of geometric fits. Thus, these results bolster the superiority of geometric fits over the algebraic approaches in terms of computational accuracy which can be seen from the summarized scores shown in Table 3. If one is ready to forsake the accuracy of error estimation to gain the benefit of lesser computation time, algebraic fits may be safely used for CMM data. Use of algebraic fits for form data is not suggested due to the marked drop in performance in contrast to the data from CMM. Among the algebraic fits, Kasa circle is recommended for use as it takes approximately one-tenth of the computation time required for one iteration of the Levenberg-Marquardt algorithm which is a geometric fit. Fig 2 shows the roundness plot for form data 5 used in Table 2. The radius R_{LSC} and center(O) with coordinates Cx, Cy was calculated using Landau Fit and the difference between the two radii R_{max} and R_{min} gives the required roundness error. The following are their values: Cx = -1.2795, Cy = 0.5734, R_{max} = 78.8179, R_{min} = 42.3359 and R_{LSC} = 55.6542.

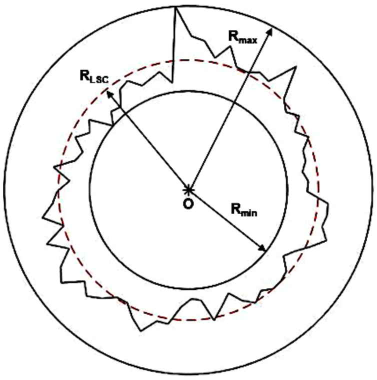


Fig 2. Roundness plot for form data 5 using Landau Fit

On comparing the difference found among the CMM and form data results, it is easy to observe that CMM results have a smaller variation amongst them. That is, the difference in magnitude among the roundness error values for a particular dataset is lesser for CMM when compared to form data. Due to the mechanical filtering caused by the CMM probe size, data acquired from CMM have a limitation in detecting undulations smaller than its diameter. Moreover, the number of representative sample data points collected from CMM is less compared to that of sample data acquired from form tester. Because of these, the CMM data acquired from the workpiece more closely approximates a circle when compared to data from the form tester. With the form test probe being smaller in size, can detect these variations more effectively, thus, making the form data comparatively more staggered. Hence, the least square algorithms do not get much scope to showcase their difference in efficiency of calculation of roundness error. This entails all of them to give very close roundness error values.

5. CONCLUSIONS

Ten different least-square algorithms were compared to quantify their relative performance in the accuracy of roundness error calculation. The ten benchmark algorithms covered both classes of geometric and algebraic fits. The study was conducted for ten data inputs, and, algorithms to be used in CMM and form testing device has been suggested independently. All the assessed least-square algorithms have very close accuracy for CMM data due to the lack of variation of the measured profile from the CMM and the ideal circle. The Levenberg-Marquardt algorithm and Trust Region Method are recommended for use as they scored the highest among the ten algorithms which were compared. Landau Fit and Spath Circle are forwarded for form data. In cases where computational time has precedence over accuracy, Kasa circle (algebraic fit) may be used. It is computationally quick and roundness error can be found without compromising much on accuracy. This comparative study based on LSC methods not only establishes the best approach for roundness error computation using CMM/form tester for academic community but is also of use to metrological system designers to make an optimum choice of algorithm to be implemented.

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